

# AN INTRODUCTION TO NONLINEAR LIOUVILLE THEOREMS FOR REACTION-DIFFUSION EQUATIONS AND SYSTEMS AND THEIR APPLICATIONS

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ABSTRACT. Liouville type theorems have now become a very central tool in the qualitative study of (elliptic and parabolic) reaction-diffusion equations and systems. Such problems include the classical nonlinear heat equation  $u_t - \Delta u = u^p$ , as well as various systems arising in applications, such as Gross-Pitaevskii or Lotka-Volterra type systems. Liouville type theorems are especially useful for the derivation of a priori estimates and their numerous applications and for the analysis of singularity formation phenomena. It is a fast growing field with already many strong results as well as important open problems. This course is intended to be just an introduction to this vast subject.

Tentative program:

- Liouville theorems for elliptic equations and systems: Gidas-Spruck by integral estimates, moving planes, Pohozaev-interpolation-feedback arguments.
  - Liouville theorems for elliptic equations and systems: Bidaut-Vron technique, zero number, rescaled energy, Merle-Zaag theorem and applications.
  - Some applications: Final and initial blowup estimates, a priori estimates, universal bounds. Some tools: doubling lemma, rescaling techniques.
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